The ETF Tangency Portfolio

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Overview

The Factor Zoo

Many factors and anomalies with positive alpha exist!

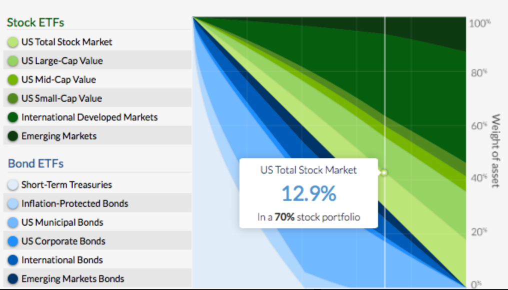
- Size
- Value
- Momentum
- Short and long term reversal
- Betting against β
- Direct profitability
- Dividend initiation
- ...

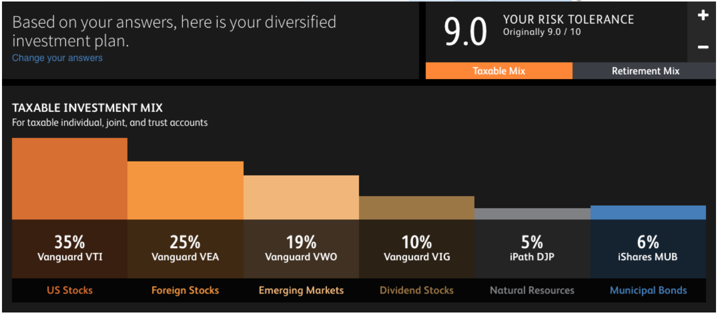
Investor's Dilemma

How can I access these unattainable factor returns?

Is there an *optimal* way to allocate among passive ETF's?

Investors Desire Advice





Opportunities for Improvement

- **Ad-hoc selection of assets**
- Highly constrained optimization
- Unclear exposure of investor's portfolio

Our Contribution

- Model unattainable (target) returns via ETF **factor models**
- Develop algorithm to **select** ETF factors
- Provide an optimal **portfolio** of a small number ETF's

Algorithm for ETF Selection

1. Sample ETF's via Matrix-Variate SSVS
2. Calculate a model-implied optimal portfolio
3. Loss function selection of ETF's using sampled optimal portfolio (similar to Hahn and Carvalho, *JASA* 2015)

An ETF-APT Model

- **Target Assets:** $\{R_j\}_{j=1}^{q}$
- **ETF Factors:** $\{ETF_i\}_{i=1}^{p}$

$$R_j = \beta_{j1} ETF_1 + \cdots + \beta_{jp} ETF_p + \epsilon_j$$

$$\epsilon_j \sim N(0, \sigma^2)$$

Sampling the Model

- Matrix-Variate SSVS:

  $M_\gamma : \quad \mathbf{R} \sim \mathrm{MN}_{N,q}\left(\mathbf{E}_\gamma \boldsymbol{\beta}_\gamma,\ \sigma^2 \mathbb{I}_{N\times N},\ \mathbb{I}_{q\times q}\right)$

- Prior on σ and β: g-priors (Empirical Bayes)

- Prior on model space: $\mathbf{P}\left(M_\gamma\right)$ (Uniform $\left(\frac{1}{2^p}\right)$ or Multiplicity Adjusted $\left(\frac{1}{p+1}\frac{1}{\binom{p}{k_\alpha}}\right)$)

Implied Optimal Portfolio

- Model implied moments

$$\mu_R = \mathbb{E}[\mathbf{R}] = \mu_{E_\gamma}^T \boldsymbol{\beta}_\gamma$$
$$\Sigma_R = var[\mathbf{R}] = \boldsymbol{\beta}_\gamma \Sigma_{E_\gamma} \boldsymbol{\beta}_\gamma^T + \Psi$$

- Optimal weights

$$w_R^* \propto \mu_R^T \Sigma_R^{-1}$$

- Optimal portfolio return

$$y_R^* = {w_R^*}^T \mathbf{R}$$

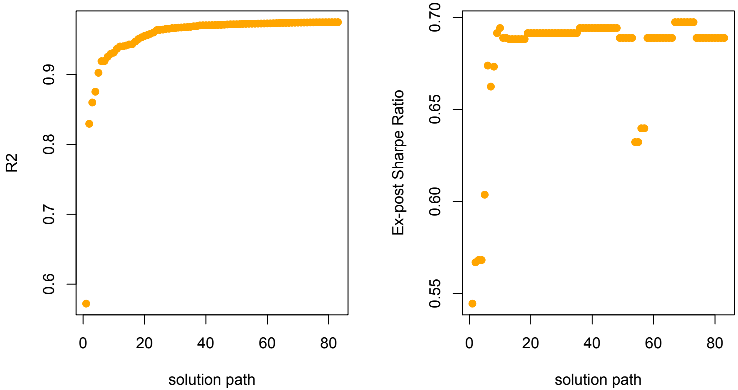
Selection via a Loss Function

- For each MCMC draw, save implied optimal portfolio
- $\bar{y}$: point-wise mean return of sampled optimal portfolio
- $\gamma_\lambda^* = \text{argmin } \|\bar{y} - \mathbf{E}\gamma\|_2^2 + \lambda\|\gamma\|_1$ with $\gamma \geq 0$
- **ETF portfolio** defined by sparse optimal weight vector: γ_λ^*

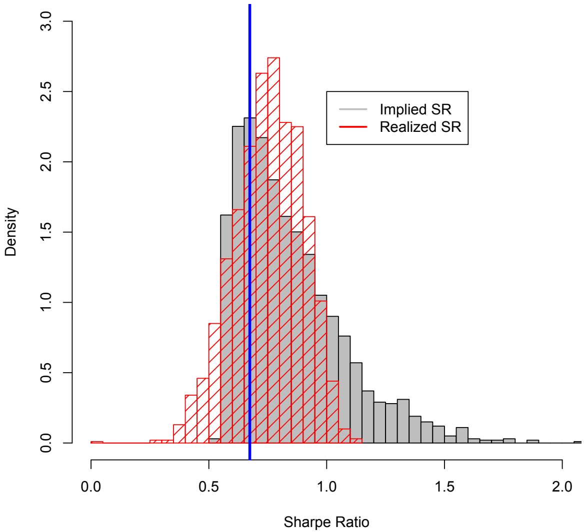
An Example (2003-2013)

- **Target assets**: Fama-French five, long and short term reversal, momentum
- **ETF's**: top 46 most liquid equity ETF's

Solution Path

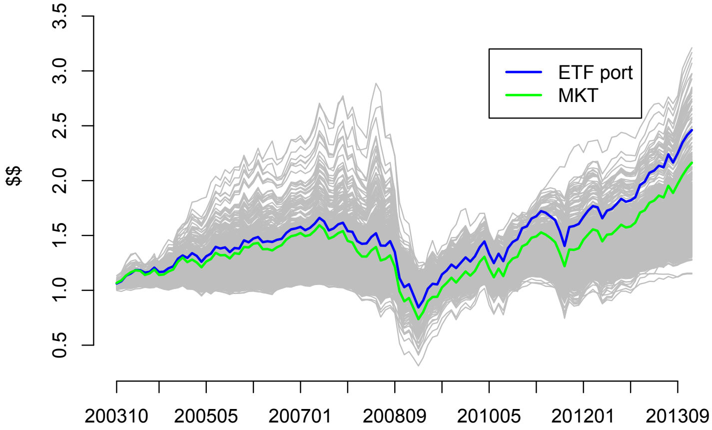


Sampled Sharpe Ratios - implied optimal portfolio



Selected Portfolio

ETF	IWD	IJR	IVW	XLE	XLP
weight	28.5%	31.2%	10.9%	10.4%	19%



Many Extensions

- Choosing different target assets
- Mutual Fund benchmarking
- DSS Loss Function: $\gamma^*_\lambda = \text{argmin } \|\mathbf{E}\bar{\beta} - \mathbf{E}\gamma\|_2^2 + \lambda\|\gamma\|_1$

  (Hahn and Carvalho, *Decoupling shrinkage and selection in Bayesian linear models: a posterior summary perspective*, JASA 2015)

Thanks!

A: Importance of APT assumption

- Errors uncorrelated across test assets
- $M_\gamma : \quad \mathbf{R} \sim \mathrm{MN}_{N,q}\left(\alpha + \mathbf{E}_\gamma \boldsymbol{\beta}_\gamma,\ \sigma^2 \mathbb{I}_{N\times N},\ \mathbb{I}_{q\times q}\right)$

$$\Longrightarrow$$

$$m_\gamma(\mathbf{R}) = \int \Pi_{i=1}^{q} \mathrm{N}_N\left(\mathbf{R} \mid \alpha^i + \mathbf{E}_\gamma \boldsymbol{\beta}^i_\gamma,\ \sigma^2 \mathbb{I}_{N\times N}\right) * \pi^i_\gamma\left(\alpha^i, \boldsymbol{\beta}^i_\gamma, \sigma\right) d\alpha^i d\boldsymbol{\beta}^i_\gamma d\sigma$$

$$= m_\gamma\left(\mathbf{R}^f_1\right) \times \cdots \times m_\gamma\left(\mathbf{R}^f_q\right)$$

$$= \Pi_{i=1}^{q} m_\gamma\left(\mathbf{R}^f_i\right)$$

A. The Search Algorithm for ETF Selection

1. Calculate Bayes Factors of two models:

$$\gamma_a = (\gamma_1, \cdots, \gamma_{i-1}, 1, \gamma_{i+1}, \cdots, \gamma_p)$$
$$\gamma_b = (\gamma_1, \cdots, \gamma_{i-1}, 0, \gamma_{i+1}, \cdots, \gamma_p)$$

2. Sample Model Parameters
3. Calculate Inclusion Probabilities via Gibbs Sampler

A: Prior on α^i, σ, $\boldsymbol{\beta}^i_\gamma$

$$\pi^i_\gamma \left(\alpha^i, \boldsymbol{\beta}^i_\gamma, \sigma \mid g^i_\gamma\right) = \sigma^{-1} \mathrm{N}_{k_\alpha} \left(\boldsymbol{\beta}^i_\gamma \mid \mathbf{0}, g^i_\gamma \sigma^2 (\mathbf{X}^T_\gamma \mathbf{X}_\gamma)^{-1}\right)$$

$$\Longrightarrow$$

$$B_{\gamma 0} = \Pi_{i=1}^{p} \frac{\left(1 + g^i_\gamma\right)^{(N-k_\gamma-1)/2}}{\left(1 + g^i_\gamma \frac{SSE^i_\gamma}{SSE^i_0}\right)^{(N-1)/2}}$$

A: Gibbs Sampler

1. Choose column $\mathbf{Y}^{rot(i)}$ and consider two models γ_a and γ_b:

$$\gamma_a = (\gamma_1, \cdots, \gamma_{i-1}, 1, \gamma_{i+1}, \cdots, \gamma_p)$$
$$\gamma_b = (\gamma_1, \cdots, \gamma_{i-1}, 0, \gamma_{i+1}, \cdots, \gamma_p)$$

2. For each model, calculate B_{a0} and B_{b0}.
3. Sample

$$\gamma_i \mid \gamma_1, \cdots, \gamma_{i-1}, \gamma_{i+1}, \cdots, \gamma_p \sim Ber(p_i)$$

where:

$$p_i = \frac{B_{a0}\mathbf{P}\left(M_{\gamma_a}\right)}{B_{a0}\mathbf{P}\left(M_{\gamma_a}\right) + B_{b0}\mathbf{P}\left(M_{\gamma_b}\right)}$$